**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer:

Here,

Standard deviation=8 Mean=45+10=55 X=60

Code:

# Normal distribution

from scipy.stats import norm

N=norm(55,8)

# P(x>60)

1-N.cdf(60)

Output : 0.26599

Option B is the Correct answer

So the probability that the service manager cannot meet his commitment is : 0.26599

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.

Answer:

Here,

N=400 Mean=38 Standard deviation=6

1. More employees at the processing center are older than 44 than between 38 and 44.

Code 1:

from scipy.stats import norm

N=norm(38,6)

#P(x>44)

1-N.cdf(44)

Out put : 0.1586

Such that (15.8/100)\*400 = 63 people out of 400

Code 2:

from scipy.stats import norm

N=norm(38,6)

#P(38-44)

N.cdf(44)-N.cdf(38)

Out put: 0.3413

Such that (34.1/100)\*400 = 137 people out of 400

So,

Given condition is False (137>63)

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Code:

from scipy.stats import norm

N=norm(38,6)

#P(x<30)

N.cdf(30)

Out put: 0.0912

Such that (9.1/100)\*400 = 36 people out of 400

So,

Given condition is True

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Anwser:

We know that ,

X 1 ~ N(μ, σ2 ) and X2 ~ N(μ, σ2 ) are following Normal distribution with Mean = μ and

Standard deviation = σ.

The mean of 2X1 wil be 2μ and,

The variance of 2X1 will be 4σ2

Therefore, 2X1 ~ N(2 µ , 4 σ2 )

Similarly,

The mean of X1 + X2 wil be 2μ and,

The variance of X1 + X2 will be 2σ2

Therefore, X1 + X2~ N(2 µ , 2 σ2 )

So,

The difference between 2X1 and X1+X2 is

* 2X1 - (X1+X2) = N( 0,2σ2 )

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answer:

Given,

Here a and b are symmetric about mean

Hence , for 99% , we need to add 0.5% on either side = 0.99+0.005 = 0.995

Code 1:

from scipy.stats import norm

z\_value = norm.ppf(0.005)

Out put : Z value of 0.005 = -2.57

Code 2:

from scipy.stats import norm

z\_value = norm.ppf(0.995)

Out put : Z value of 0.995 = 2.57

We know the Formula ,

Z = x-Mean/SD

x = SD \* Z + Mean

x = 20\*Z + 100

Therefore ,

a = (20 \* (-2.57)) + 100

a = 48.5

b = (20\* 2.57) + 100

b = 151.5

So, Correct option is (48.5, 151.5 ) (D)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

Answer :

Given,

Profit1 ~ N(5, 32) , Profit2 ~ N(7, 42)

Total profit ~ N(12, 52)

Convert into Rupees,

$1 = Rs. 45

Mean Profit is 12\*45 = Rs 540 Million

Standard Deviation is 5\*45 = Rs 225.0 Million

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Code :

from scipy.stats import norm

stats.norm.interval(0.95, loc = 540, scale = 225)

Out put : (99.008, 980.991)

The range is (99,980) where the probability of 95% of the annual profit of the

company contains

1. Specify the 5th percentile of profit (in Rupees) for the company

we know that ,

Z value for 5th percentile is = -1.645

X = SD\*Z + Mean is ,

X= 540+(-1.645)\*(225)

X= 170

So,

5th percentile of profit is 170 (in Million Rupees)

1. Which of the two divisions has a larger probability of making a loss in a given year?

Making loss, Means X<0

Division 1 :

Code :

from scipy.stats import norm

N=norm(5,3)

#P(x<0)

N.cdf(0)

Output : 0.04779035

Division 2 :

Code :

from scipy.stats import norm

N=norm(7,4)

#P(x<0)

N.cdf(0)

Output : 0.04005915

Hence ,

Division 1 will face more loss.